ASTRON 429, Fall 2017 – Independent Study

Due on Friday Dec. 1st, by 5 PM in Alex Gurvich's mailbox.

This extra problem set for students registered for the graduate version of the course goes beyond the chapters that we had time to cover during the quarter by exploring some basic topics in cosmological structure formation and galaxy evolution.

Read chapters 11 and 12 in Ryden, and solve the following problems. The Roman numerals below roughly divide the problems by theme. As usual, points will be given roughly in proportion to the work needed to solve each problem. Note that IV below will require you to solve equations numerically and produce plots, so you should allocate time appropriately.

I. Growth factor of linear perturbations. Starting from equation (11.49) in the textbook, re-derive (showing all your steps) the functional form for the linear evolution of matter density perturbations, $\delta(t)$, for the cases of a matter-dominated universe, a radiation-dominated universe, and a cosmological constant-dominated universe (three eras actually experienced by our universe). These results are given in the textbook; the exercise here is for you to go through the math yourself.

Then, solve problem 11.2 in Ryden, which asks you to derive the evolution of $\delta(t)$ for the case of an empty, negatively curved universe.

II. Ryden problem 11.4. This problem provides a bound on the maximum redshift at which galaxies with certain properties may be found.

III. Ryden problems 12.1 and 12.2. These will give you practice calculating quantities from the Schechter function, which is often used to characterize the galaxy luminosity function.

IV. Cosmological evolution of halos and their galaxies. Look up the following paper, https://arxiv.org/abs/1205.5807 (Moster et al. 2013), which contains a popular fit to the $M_\star - M_\mathrm{h}$ relation inferred empirically using the abundance matching technique (the most massive galaxies belong to the most massive dark matter halos, etc.). Assume a standard ΛCDM cosmology (e.g., with the parameters of the "benchmark" model in Ryden) and define halos to enclose 200 times the critical density (i.e., $M_\mathrm{h} = M_{200\mathrm{c}}$ and $R_\mathrm{vir} = R_{200\mathrm{c}}$).

To answer the following questions, you will need to write a computer code to implement the Moster et al. $M_\star - M_\mathrm{h}$ relation and to solve for its inverse. In the paper, the parameters of the fitting function are summarized in Table 1 (see equations 11-14). For the plots, take the time to choose axes carefully so as to make the plots most useful (log vs. linear, choice of units, etc.). Clearly label axes and include legends to identify different curves.

a) Make a plot of $M_\star$ vs. M_h. Assume that the fitting parameters can be extrapolated to arbitrarily high redshift and plot the relationship for $z = 0,\ 1,\ 2,\ 3,\ 4,\ 5$ and 6 (on the same panel, one curve per redshift).

b) For stellar masses $M_\star = 10^4,\ 10^6,\ 10^8$, and 10^{10} $M_\odot$, plot M_h vs. z from $z = 0$ to 6 (on the same panel, one curve per $M_\star$ value).

c) Assume (as is a fair approximation to observations) that the half-mass radius of the stellar distribution of galaxies is a constant fraction of the virial radius of the host halo, specifically $r_{1/2} = 0.02R_\mathrm{200c}$. For the same stellar masses and redshift range as in part b), plot $r_{1/2}$ vs. z.

d) For any halo, define a circular velocity $v_\mathrm{c,h} \equiv (GM_\mathrm{200c}/R_\mathrm{200c})^{1/2}$. For galaxies with flat rotation curves, the circular velocity of stars in a galaxy will be $v_{\mathrm{c},\star} \approx v_\mathrm{c,h}$. Under this approximation, show analytically that the orbital period of stars at the half-mass radius, $t_\mathrm{orb} \equiv 2\pi r_{1/2}/v_{\mathrm{c},\star}$, is a function of redshift but is independent of galaxy mass.

e) Plot t_orb vs. z for the same stellar masses and redshift range as in parts b) and c).

Using the above results, what can you conclude regarding the typical sizes and orbital periods of high-redshift galaxies relative to local galaxies? The Hubble Space Telescope (and its successor, the James Webb Space Telescope, scheduled for launch next year) is capable of observing galaxies out to $z \approx 6 - 10$. The above results enable you to anticipate how the properties of early galaxies should differ from the properties of more nearby galaxies, including the Milky Way.

V. Ryden problem 12.4. For this problem, you should do all the calculations yourself, but note that this was the subject of an actual research article (https://arxiv.org/abs/astro-ph/0302506), which so far has attracted more than 1,400 citations.